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Treatment of Geodetic Leveling in the Integrated Geodesy Approach

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Abstract and Introduction

Integrated geodesy is a method in which a wide variety of surveying measurements are modeled in terms of geometric positions and the earth's geopotential. Using heterogeneous data, both geometric and gravimetric quantities are simultaneously estimated by a least-squares procedure. Heretofore, geodetic leveling differences have been reduced into pseudo-observables using assumed values of gravity prior to their inclusion into integrated geodesy least-squares adjustments. This study compares the errors in estimates of geometric and gravimetric quantities obtained from integrated geodesy adjustments of geodetic leveling difference, potential differences and Helmert height differences.

Model

If one corrects for atmospheric and instrumental effects, then the lines of sight of a rotatable level describe a plane in space which is normal to the direction of the local gravity vector. This plane can be considered to pass through a point midway along a chord between the bases of the level rods. The level rods are aligned along their own local verticals. These local verticals need not be parallel or possess any special relationships to the local vertical at the level instrument. (In practice, the verticals will be nearly parallel). One may compute a directed distance between the base of a given level rod and the point of intersection of that level rod with the level instrument normal plane. The geodetic level difference is modeled as the difference between the directed distances at the two level rods (BD - AC in Figure 1). A detailed derivation of the geodetic level difference model can be found in Milbert [1988].

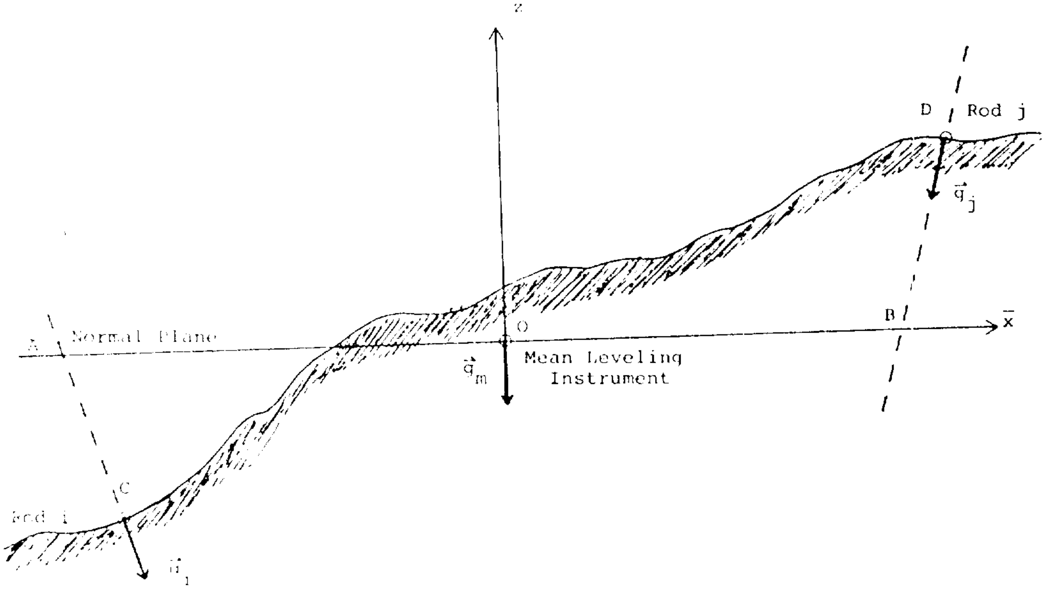

Figure 1. Side View of Level Rods and Mean Normal Plane Relationships.

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Computational Procedure

To evaluate the geodetic level difference model, and to compare its behavior to that of potential differences and Helmert height differences, a simulation approach is chosen. An analytic model (a Molodensky mountain) provides prior values of geometric and gravimetric quantities, including gravity, GPS ellipsoidal heights, and level measurements. The geometric part of the model is a conical mountain, one kilometer (km) high, with a base of about 40 km radius, resting on a spherical earth of 6369.4 km. The gravimetric part of the model is composed of a single disturbing point mass imbedded 4 km beneath the spherical earth and the OSU86F geopotential. This combination provides a non-isotropic gravity field that is more realistic than those found in other analytic models. Through the analytic model, 400 geodetic level differences (corresponding to a 40 km level route from the peak of the mountain to the base), 144 gravity measurements arranged in a $1° \times 1°$ grid, and 31 gravity measurements along the level route are obtained. With the exception of one benchmark at the peak of the mountain, the locations of the gravity measurements are not coincident with the benchmarks. In addition to the gravity measurements discussed above, derived data, which correspond to the pseudo-observables, are formed in a process consistent with that found in practice. Derived gravity values are predicted at benchmarks by collocation. Potential differences and Helmert height differences are then derived from the geodetic level differences and gravity interpolated from those values predicted at the benchmarks.

Results

As a baseline example, an integrated geodesy least-squares adjustment was computed using the 175 gravity measurements and the 40 derived potential differences. The ellipsoidal heights of the gravity stations were held fixed at the analytic model values. The ellipsoidal height of the benchmark at the peak of the mountain was fixed to eliminate a datum defect. The integrated geodesy adjustment estimates geometric position and the geopotential field and its derivatives. Figure 2 displays the error in the estimates of ellipsoidal height at the benchmarks using the model data near Denver, Colorado, potential difference pseudo-observables, and the OSU86F model in a "remove/restore" process. The errors are in the sense of estimate minus analytic model. Estimation error is induced by the disturbing point mass, which is not parameterized by the observation equations or the remove/restore process.

The integrated adjustments were repeated using either potential difference, ΔW, Helmert height difference, ΔH, or geodetic level difference data, Δn. In the case of the geodetic level differences, the measurements were fed directly into the adjustment, without any need for reduction in a pre-adjustment computation. The results of these adjustments are virtually identical to those of Figure 2. To illustrate the slight changes, Figure 3 displays differences formed when the adjustment errors of the geodetic level difference model are subtracted from the adjustment errors of the remaining models. Discrepancies due to choice of model are seen to be smaller than the measurement noise of leveling. The upper curve demonstrates that the geodetic level difference model is as effective as the potential difference model.

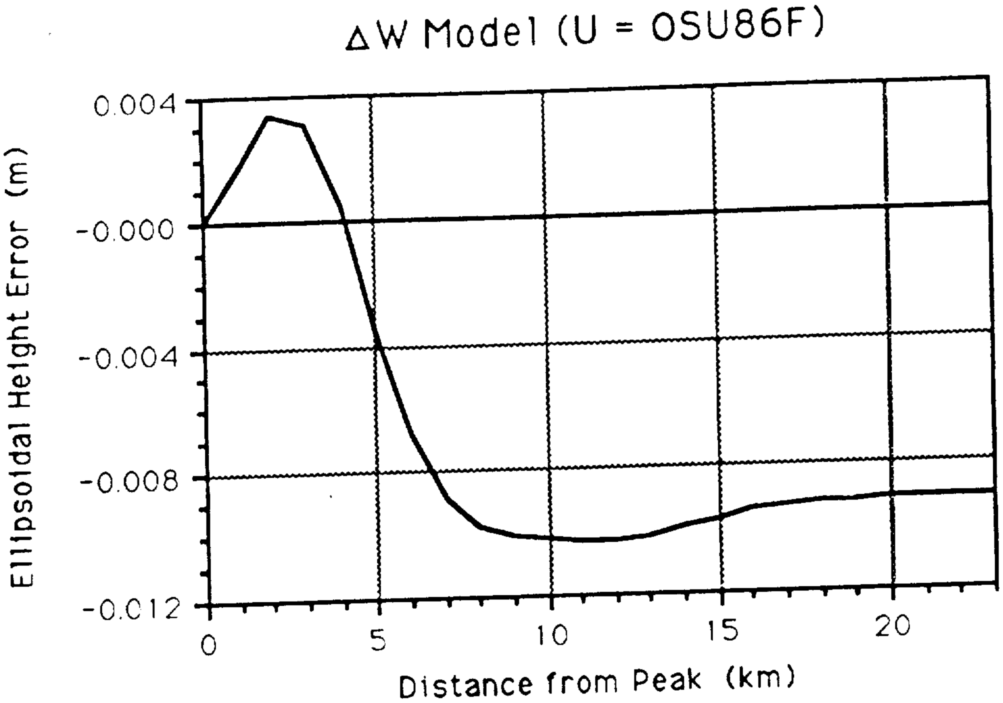

Figure 2. Error in Ellipsoidal Height Estimates at Benchmarks, Potential Difference Model, ΔW.

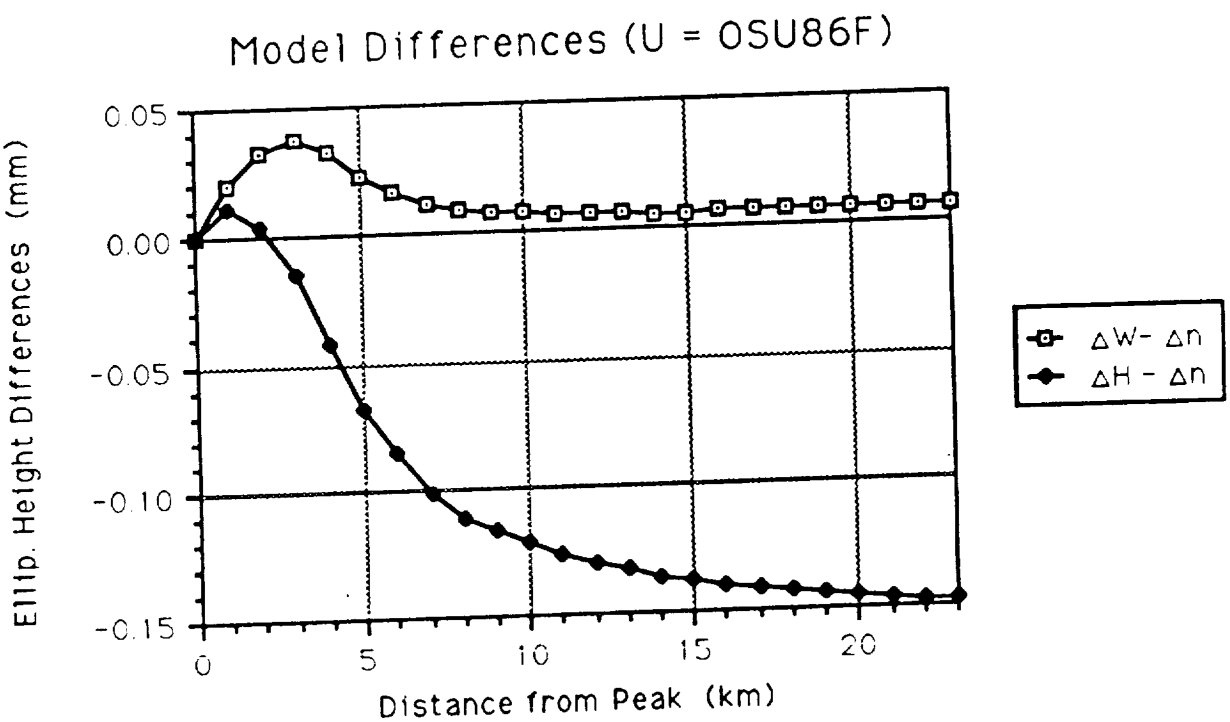

Figure 3. Ellipsoidal Height Errors Compared Between Models
(ΔW model errors - Δn model errors)
(ΔH model errors - Δn model errors)

These results were seen to hold throughout a variety of scenarios. Integrated adjustments were computed using GRS80 in place of OSU86F for linearization of the models. The influence of ellipsoidal height difference data, such as obtained from GPS signals, was examined. The analytic model was varied with regard to choice of region and magnitude of the disturbing point mass. Tests were performed to observe the influence of the gravity grid data set. And, the effects of various computational approximations to the observation equations were explored. Greater detail on these tests can be found in Milbert [1988].

Covariance Models

Since a least-squares collocation method was selected to solve the integrated geodesy observation equations, it was necessary to develop a model for the covariances and cross-covariances of components of the disturbing potential. One component of the gravimetric part of the analytic model was a high degree spherical harmonic expansion, OSU86F, complete to degree and order 360. The associated covariance model is based on those potential degree variances. The other component of the gravimetric part of the analytic model was a disturbing point mass. However, the spectrum of the potential degree variances from a point mass generates covariance functions which do not lend themselves to evaluation by the closed forms of Tscherning and Rapp [1974]. A new family of covariance functions (one member of which contains the point mass spectrum) is defined. It is shown that the covariance and cross-covariance functions for a point mass can be expressed in closed formulas by means of incomplete elliptic integrals of the first and second kind. Highly efficient algorithms exist for the evaluation of the elliptic integral functions, allowing rapid computation of the point mass covariance functions.

Conclusions

It has been found possible to model geodetic level differences in an integrated geodesy approach. By means of this model, it is not necessary to reduce geodetic level differences into potential differences or Helmert height differences in a preliminary computation.

References

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